(1.) $\langle \bigcirc\ \bigcirc \rangle = (-A^2 - A^{-2})\langle \bigcirc \rangle$

$= -A^2 - A^{-2}$

(2.) $\langle \text{trefoil} \rangle = A\langle \text{Hopf link} \rangle + A^{-1}\langle \text{unknot} \rangle$

$= A(-A^2 - A^{-2}) + A^{-1}$ (from 1.)

$= -A^3 - A^{-1} + A^{-1} = -A^3$

(3.) $\langle \text{unknot} \rangle = A\langle \text{unknot} \rangle + A^{-1}\langle \text{unlink} \rangle$

$= A \cdot 1 + A^{-1}(-A^2 - A^{-2})$ (from 1.)

$= A - A - A^{-3} = -A^{-3}$

(4.) The Hopf Link

$\langle \text{Hopf link} \rangle = A\langle \text{unlink} \rangle + A^{-1}\langle \text{unknot} \rangle$

$= A(-A^3) + A^{-1}(-A^{-3})$ (from 2 & 3)

$= -A^4 - A^{-4}$

(5.) $\langle \text{figure-8} \rangle = A\langle \text{unlink} \rangle + A^{-1}\langle \text{Hopf link} \rangle$

$= A(-A^2 - A^{-2})\langle \text{unknot} \rangle + A^{-1}\langle \text{unknot} \rangle$

$= \left[A(-A^2 - A^{-2}) + A^{-1}\right]\langle \text{unknot} \rangle$

$= \left[-A^3 - A^{-1} + A^{-1}\right](-A^3)$ (from 2.)

$= A^6$

(6.) The Left-Handed Trefoil

$\langle \text{left trefoil} \rangle = A\langle \text{figure-8} \rangle + A^{-1}\langle \text{Hopf link} \rangle$

$= A(A^6) + A^{-1}(-A^4 - A^{-4})$ (from 4 & 5)

$= A^7 - A^3 - A^{-5}$